A story of 35 years of karma

The years following my PhD have been full of drama. While I was working on the exciting project of topologically classifying real algebraic sets with Henry King, I started working with Rob Kirby, on the problem of deciding whether *Cappell-Shaneson homotopy* 4*-spheres* Σ_n $(n = 0, 1, 2..)$ are standard 4-spheres. They're the 2-fold covering spaces of some exotic copies of RP^4, which Cappell-Shaneson had constructed previously. One day Rob and I worked all night to arrive to Figure 14.1 of [A], which implied that Σ_0 is a Gluck twisted S^4, that is $\Sigma_0 = (S^4 - C) \cup_f C$, where C is a tubular neighborhood of some imbedded $S^2 \hookrightarrow S^4$ and $f : S^2 \times S^1 \to S^2 \times S^1$ is the nontrivial diffeomorphism ([Ai]). Following 2-years I improved this result extending Figure 14.1 to Figure 14.14 of [A], which is a handlebody of Σ_0 without 3-handles! This implied Σ_0 is homeomorphic to S^4! But Mike Freedman's proof of 4 dimensional topological Poincare Conjecture that time, took away the luster from my result.

I was still hoping to continue working on this project. But it looked like my coauthor Kirby has lost his apatite to continue working on this problem. He was giving talks offering Figure 14.14 as possible exotic S^4, and I was under pressure to go back to Turkiye to do my 3 months of obligatory military service. Before departing, I practically begged him not to publish the pictures Figure 14.1 through Figure 14.14 of [A], wait until I come back. But he overruled my objection (he is my elder and the teacher). As a last resort, I staged a mini-protest of refusing to draw these pictures for the journal. But he hired an artist to draw them, and proceeded to published it as "A possible counterexample to smooth Poincare Conjecture". I then swallowed my pride and left for Turkiye.

I must admit, sometime after that, when I saw the paper by [G] I almost had an heart attack :) My younger academic brother Bob Gompf was proving Σ_0 is standard (by using my pictures). I could not criticize Gompf for his advisors callousness. I picked myself up, and vowed never to think about C-S spheres again... But 20 years later, one day in Gokova. Sema Salur walked into my office to show me a newly posted paper [FGMW]. This paper was claiming that the remaining C-S spheres, Σ_n with $n > 0$ will be exotic, up to checking hundredths of hours of Microsoft computers round the clock calculations. This time I did not have a heart attack, I just stayed up all night to prove they are wrong, in fact all the homotopy spheres Σ_n for all n, are diffeomorphic to S^4 and I posted my paper [AK3] next day. Thank god I lived long enough to see the end of this story (this story reminds me the more recent story of [MP]).

Epilogue: If you ask me "In the beginning you started praising your advisor, now you sound critical, what happened?" My answer is "Time happened!" This story has 50 years of time span. I don't know why people change by time, but I do know people don't like to be reminded of their shortcomings, if you do you might be disliked as an outcast (these days this is called "getting cancelled"). It's always better to stand alone with the truth, than to be popular in a crowd.

References

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